## AS Level Mathematics A <br> H230/01 Pure Mathematics and Statistics

## Wednesday 16 May 2018 - Morning <br> Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{gm} \mathrm{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g=9.8$.


## INFORMATION

- The total mark for this paper is 75 .
- The marks for each question are shown in brackets [ ].
- You are reminded of the need for clear presentation in your answers.
- The Printed Answer Booklet consists of 16 pages. The Question Paper consists of 8 pages.


## Formulae

## AS Level Mathematics A (H230)

## Binomial series

$(a+b)^{n}=a^{n}+{ }^{n} \mathrm{C}_{1} a^{n-1} b+{ }^{n} \mathrm{C}_{2} a^{n-2} b^{2}+\ldots+{ }^{n} \mathrm{C}_{r} a^{n-r} b^{r}+\ldots+b^{n} \quad(n \in \mathbb{N})$,
where ${ }^{n} \mathrm{C}_{r}={ }_{n} \mathrm{C}_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}$

Differentiation from first principles
$\mathrm{f}^{\prime}(x)=\lim _{h \rightarrow 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

## Standard deviation

$\sqrt{\frac{\sum(x-\bar{x})^{2}}{n}}=\sqrt{\frac{\sum x^{2}}{n}-\bar{x}^{2}}$ or $\sqrt{\frac{\sum f(x-\bar{x})^{2}}{\sum f}}=\sqrt{\frac{\sum f x^{2}}{\sum f}-\bar{x}^{2}}$

The binomial distribution
If $X \sim \mathrm{~B}(n, p)$ then $P(X=x)=\binom{n}{x} p^{x}(1-p)^{n-x}$, Mean of $X$ is $n p$, Variance of $X$ is $n p(1-p)$

## Kinematics

$v=u+a t$
$s=u t+\frac{1}{2} a t^{2}$
$s=\frac{1}{2}(u+v) t$
$v^{2}=u^{2}+2 a s$
$s=v t-\frac{1}{2} a t^{2}$

## Section A: Pure Mathematics

Answer all the questions.

1 In this question you must show detailed reasoning.
(i) Express $3^{\frac{7}{2}}$ in the form $a \sqrt{b}$, where $a$ is an integer and $b$ is a prime number.
(ii) Express $\frac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c+d \sqrt{e}$, where $c$ and $d$ are integers and $e$ is a prime number.

2 (i) The equation $x^{2}+3 x+k=0$ has repeated roots. Find the value of the constant $k$.
(ii) Solve the inequality $6+x-x^{2}>0$.

3 (i) Solve the equation $\sin ^{2} \theta=0.25$ for $0^{\circ} \leqslant \theta<360^{\circ}$.
(ii) In this question you must show detailed reasoning.

Solve the equation $\tan 3 \phi=\sqrt{3}$ for $0^{\circ} \leqslant \phi<90^{\circ}$.

4 (i) It is given that $y=x^{2}+3 x$.
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(b) Find the values of $x$ for which $y$ is increasing.
(ii) Find $\int(3-4 \sqrt{x}) \mathrm{d} x$.
$5 \quad N$ is an integer that is not divisible by 3 . Prove that $N^{2}$ is of the form $3 p+1$, where $p$ is an integer.

6 Sketch the following curves.
(i) $y=\frac{2}{x}$
(ii) $y=x^{3}-6 x^{2}+9 x$
$7 O A B C$ is a parallelogram with $\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O C}=\mathbf{c} . P$ is the midpoint of $A C$.

(i) Find the following in terms of $\mathbf{a}$ and $\mathbf{c}$, simplifying your answers.
(a) $\overrightarrow{A C}$
(b) $\overrightarrow{O P}$
(ii) Hence prove that the diagonals of a parallelogram bisect one another.

## 8 In this question you must show detailed reasoning.

The lines $y=\frac{1}{2} x$ and $y=-\frac{1}{2} x$ are tangents to a circle at $(2,1)$ and $(-2,1)$ respectively. Find the equation of the circle in the form $x^{2}+y^{2}+a x+b y+c=0$, where $a, b$ and $c$ are constants.

## Section B: Statistics

Answer all the questions.

9 Jo is investigating the popularity of a certain band amongst students at her school. She decides to survey a sample of 100 students.
(i) State an advantage of using a stratified sample rather than a simple random sample.
(ii) Explain whether it would be reasonable for Jo to use her results to draw conclusions about all students in the UK.

10 The probability distribution of a random variable $X$ is given in the table.

| $x$ | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $\frac{3}{8}$ | $\frac{5}{16}$ | $4 p$ | $p$ |

(i) Find the value of $p$.
(ii) Two values of $X$ are chosen at random. Find the probability that the product of these values is 0 .

11 The probability that Janice sees a kingfisher on any particular day is 0.3 . She notes the number, $X$, of days in a week on which she sees a kingfisher.
(i) State one necessary condition for $X$ to have a binomial distribution.

Assume now that $X$ has a binomial distribution.
(ii) Find the probability that, in a week, Janice sees a kingfisher on exactly 2 days.

Each week Janice notes the number of days on which she sees a kingfisher.
(iii) Find the probability that Janice sees a kingfisher on exactly 2 days in a week during at least 4 of 6 randomly chosen weeks.

12 It is known that $20 \%$ of plants of a certain type suffer from a fungal disease, when grown under normal conditions. Some plants of this type are grown using a new method. A random sample of 250 of these plants is chosen, and it is found that 36 suffer from the disease. Test, at the $2 \%$ significance level, whether there is evidence that the new method reduces the proportion of plants which suffer from the disease.

13 The radar diagrams illustrate some population figures from the 2011 census results.


Each radius represents an age group, as follows:

| Radius | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age <br> group | $0-17$ | $18-29$ | $30-44$ | $45-59$ | $60-74$ | $75+$ |

The distance of each dot from the centre represents the number of people in the relevant age group.
(i) The scales on the two diagrams are different. State an advantage and a disadvantage of using different scales in order to make comparisons between the ages of people in these two Local Authorities.
(ii) Approximately how many people aged 45 to 59 were there in Liverpool?
(iii) State the main two differences between the age profiles of the two Local Authorities.
(iv) James makes the following claim.
"Assuming that there are no significant movements of population either into or out of the two regions, the 2021 census results are likely to show an increase in the number of children in Liverpool and a decrease in the number of children in Rutland."

Use the radar diagrams to give a justification for this claim.

## END OF QUESTION PAPER

BLANK PAGE

Oxford Cambridge and RS

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.
If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.
For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.
OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

